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Receding Horizon Following Control of Wheeled Mobile Robots: A Case Study

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***Abstract*—This paper presents a receding horizon controller for path following problem of a wheeled mobile robot. In general, a suitable terminal penalty and a terminal region are adopted to guarantee the recursive feasibility of the optimization problem and asymptotic convergence to the reference path, but they are usually difficult to be calculated online and often prepared offline. However, we can avoid this complex calculation by using the reference path as the terminal region for a special case study. Sufficient conditions on the asymptotic convergence and recursive feasibility of the proposed scheme are investigated. Subsequently, the designed controller is applied to the following problem of the wheeled mobile robot to illustrate the control performance of our approach. The effectiveness of the presented control strategy is verified by the simulation results.**

Index Terms - model predictive control; constrained nonlinear systems; mobile robots; path following.

I. INTRODUCTION

There are three fundamental motion control problems in the control field, namely the set-point stabilization, trajectory tracking and path following. Set-point stabilization, as a classical regulation problem, is to regulate the state of the system to a fixed target state [1].Trajectory tracking (TT) aims at tracking a given time varying reference trajectory, which is a time parameterized curve [2]. Recently, path following (PF) has been intensively investigated, which is regarded as an alternative problem formulation to trajectory tracking. PF is also to guarantee the system to follow a reference path, but at this moment the reference is parameterized in its geometrical coordinates. However, trajectory tracking and path following problems are based on the set-point stabilization, but to some extent they are beyond set-point stabilization problem.

In contrast to the tracking control problem, the differences between the TT and PF problem are reflected in the following aspects: First, the extent of the time requirements in tracking error is different, TT has strict requirements on time, but it is not the same case with PF problem. Second, the reference trajectory is highly dependent on the reference model in TT problem, whereas the PF system doesn't has this limitation. Finally, the reference trajectory explicitly shows that when the system should be where in the state space, but following a parameterized reference path means to design a controller which both affects the system behavior and controls reference path evolution [3]. So path following controller should determine velocity online, which is an extra degree of freedom in the controller design. In other words, the evolution of the path parameter is supposed to be obtained in the controller in order to achieve forward movement and error minimization.

Due to the advantages of the PF control scheme like the aforementioned information, which are obtained from the comparison with the TT systems, path following is widely applied in (automobile, ship or flight) course control, car-parking problem or the control of robots [4], and batch crystallization [5]. Accordingly, many different PF control schemes have been explored. For example, path following was developed by feedforward control scheme in [6], transverse feedback linearization techniques in [7], backstepping designs in [8] and sliding mode control in [9].These controllers are either restricted for application or limited in the fact that the state and input constraints are not considered.

Since receding horizon control, also referred as nonlinear model predictive control (NMPC), can handle the optimization problems for nonlinear systems subject to state and input constraints, it has become one of the most standard and frequently used techniques recently. In general, at each sampling instant, NMPC scheme solves an online finite horizon open-loop optimal algorithm based on current states and system model, and finds a control sequence by numerical optimization, then applies the first control action to the system. NMPC has achieved rich results in stability, robustness and optimality analysis, and has formed mature scientific analysis and design methods [10,11]. Many exhaustive results on NMPC for regulation problems are available. Predictive tracking control scheme has also been discussed, although quite a lot literature only considered tracking the piecewise constant references, as in [12]. Currently many researchers have applied the NMPC framework to solve the PF problems and showed the stability analysis [13-15], these schemes are either only fit for the given system or rely on some certain characteristics of system, like the property of differential flatness.

Normally, NMPC stability is achieved by adding a terminal penalty to the cost function and restricting terminal state to a terminal region. But it is usually difficult to calculate the suitable terminal ingredients, and sometimes they are time consuming, especially for nonlinear or time-varying systems. So the terminal ingredients are locally defined and generally prepared well offline, which will lead to a potentially restricted region of attraction and stay at a disadvantage. Therefore many optimization approaches for calculating the terminal region have been proposed as in [16] and references

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therein. However, [17] has proved that forcing the terminal state to equal zero can guarantee the stability for finite receding horizon. Motivated by above observations, this paper will design a receding horizon controller to achieve PF control of a wheeled mobile robot. To avoid the complex online calculations of the suitable terminal ingredients in the controller design, the main idea of the NMPC scheme for PF in this paper, is to consider the path as the terminal region.

This paper is organized as follows. The kinematics model of a nonholonomic mobile robot is described in Section Ⅱ. Section III first introduces the path following problem, then an NMPC scheme is discussed, where a method of choosing the path as the terminal region with a proof of feasibility is presented. Section IV provides the simulation implementation and results of the mobile robot. Finally, our brief summary is given in Section V.

II. Following Control Task

The nonholonomic wheeled mobile robot with a path following problem has a front castor and two rear driving wheels, which is shown in the world coordinate system composed of axes x and y, as shown in Fig.1, the corresponding symbol definitions are described in Table I.

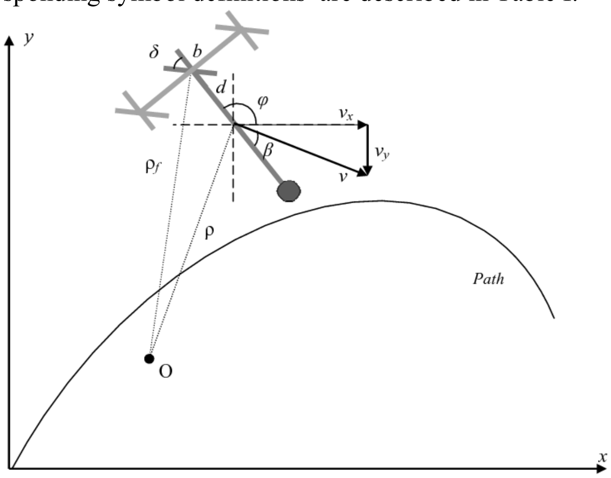

Fig.1 The simplified model of a wheeled mobile robot

TABLE I
The Description for Symbol Definitions

Parameters	Symbol
track between front wheels	$2b$
vertical distance between centroid and front wheel	d
the radius of the wheel	r
instantaneous center of robot	O
distance between instantaneous center and the front wheel	ρ_f
distance between instantaneous center and the centroid	ρ
resultant velocity of the centroid	v
side slip angle	β
yaw angle	φ
steering angle	δ
the velocity of the left wheel	w_l
the velocity of the right wheel	w_r

So according to the geometric relationship as shown in Fig.1

$$\rho_f = \frac{2b}{\omega_r - \omega_l}\frac{\omega_r + \omega_l}{2} = \frac{\omega_r + \omega_l}{\omega_r - \omega_l} b \tag{1}$$

$$\rho = \sqrt{(\rho_f \sin\delta - d)^2 + (\rho_f \cos\delta)^2} \tag{2}$$

The yaw rate of the wheeled mobile robot is given by

$$\dot{\varphi} = \frac{\frac{\omega_r + \omega_l}{2} r}{\rho_f} = \frac{r}{2b}(\omega_r - \omega_l) \tag{3}$$

Thus

$$v = \rho\dot{\varphi} = \frac{r}{2b}(\omega_r - \omega_l)\sqrt{(\rho_f \sin\delta - d)^2 + (\rho_f \cos\delta)^2} \tag{4}$$

The motion state of the robot can be described by vector $\mathrm{X} = (x, y, \varphi)^T$, which includes its position (x, y) and orientation φ. Therefore, the kinematics equation when the tire deformation is neglected is as follows

$$\dot{\mathrm{X}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} v\cos(\beta + \varphi) \\ v\sin(\beta + \varphi) \\ r\cdot(\omega_r - \omega_l)/2b \end{bmatrix} \tag{5}$$

where $\beta = \arctan\dfrac{\rho_f \sin\delta - d}{\rho_f \cos\delta}$.

From (4) we can see that (5) is an extremely complex nonlinear equation. We take a special example to verify the kinematics equation (5), when $\delta = 0$

$$\begin{cases} \sin\beta = d/\rho \\ \cos\beta = \rho_f/\rho \\ v = \rho r(\omega_r + \omega_l)/2\rho_f \end{cases} \tag{6}$$

By means of trigonometric formulas and (6), thus

$$\begin{aligned} \dot{x} &= \frac{\cos\varphi}{2} r(\omega_r + \omega_l) - \frac{\sin\varphi}{2b} rd(\omega_r - \omega_l) \\ \dot{y} &= \frac{\sin\varphi}{2} r(\omega_r + \omega_l) - \frac{\cos\varphi}{2b} rd(\omega_r - \omega_l) \\ \dot{\varphi} &= \frac{r}{2b}(\omega_r - \omega_l) \end{aligned} \tag{7}$$

Equation (7) is the same as the kinematics equation of the mobile robot without steering in [18].

Equation (5) is too complex, we need to simplify it for designing the controller. Particularly, we choose specific δ such that

$$\rho_f \sin\delta - d = 0 \tag{8}$$

So (5) can be transformed into

$$\dot{\mathrm{X}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} v\cos\varphi \\ v\sin\varphi \\ w \end{bmatrix} \tag{9}$$

where $\beta = 0, v = \frac{r}{2}(\omega_r + \omega_l)\cos\delta, w = \frac{r}{2b}(\omega_r - \omega_l)$. w can be seen as angular velocity of the yaw angle for brief description.

Remark 1 $\beta = 0$ in (9) is favourable for stability control. Besides, there is not any singular point when $\omega_r - \omega_l = 0$.

To depict the error state, we assume that there is a virtual mobile robot moving along the reference path, its position and orientation represent the ideal movement state. So the current reference path needs to be described by a reference state $\mathrm{X}_R = (x_R, y_R, \varphi_R)^T$.

Accordingly

$$\dot{\mathrm{X}}_R = \begin{bmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{\varphi}_R \end{bmatrix} = \begin{bmatrix} v_R \cos\varphi_R \\ v_R \sin\varphi_R \\ w_R \end{bmatrix} \tag{10}$$

so the error state $\mathrm{X}_e = (x_e, y_e, \varphi_e)^T$, which denotes the robot current position with respect to the reference, under the virtual reference robot coordinate system is as follows

$$\mathrm{X}_e = \begin{bmatrix} x_e \\ y_e \\ \varphi_e \end{bmatrix} = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_R - x \\ y_R - y \\ \varphi_R - \varphi \end{bmatrix} \tag{11}$$

Computing the derivatives of (11) and some calculation, we can derive the error dynamic model

$$\dot{\mathrm{X}}_e = \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\varphi}_e \end{bmatrix} = \begin{bmatrix} w y_e - v + v_R \cos\varphi_e \\ -w x_e + v_R \sin\varphi_e \\ w_R - w \end{bmatrix} \tag{12}$$

It is worth mentioning that when v, w approaches v_R, w_R respectively, X_e is able to stay on the equilibrium $\mathbf{0}$ from (12).

Based on the error state X_e and its dynamic model (12), the path following control task of a wheeled mobile robot can be concluded as: Given a reference path, find admissible control law $u = (v, w)^T$ to drive the error state X_e to zero.

Remark 2 Sometimes the control signal in the path following control of mobile robot is chosen as $u' = (w_l, w_r, \delta)^T$ from (5). The relationship between u and u' is given by using (8)

$$\begin{cases} r(\omega_r + \omega_l)\cos\delta = 2 \cdot v \\ r(\omega_r - \omega_l) = 2b \cdot w \\ \dfrac{(\omega_r - \omega_l)}{(\omega_r + \omega_l)} \dfrac{d}{b} = \sin\delta \end{cases} \tag{13}$$

where r, b, d are the given parameters of the wheeled mobile robot according to TABLE I.

III. Receding Horizon Following Control

Consider a continuous time nonlinear nominal system of the form

$$\dot{x}(t) = f(x(t), u(t)), \quad x(t_0) = x_0 \tag{14}$$

where $x \in X \subseteq R^n$ and $u \in U \subseteq R^m$ are state and input constraints. The nonlinear function $f(x,u): X \times U \to R^n$ is continuous and locally Lipchitz respect to x.

The objective of path following is to make system state $x(t)$ follow a parameterized reference path, which is defined in the state space by a scalar θ

$$P = \left\{ r \in R^n \middle| r = p(\theta) \right\} \tag{15}$$

The map $p: R^1 \to R^n$ is a twice continuously differentiable function. The scalar $\theta \in \hat{\theta} \subseteq R^1$ is not given a priori but specified by a virtual input

$$\dot{\theta}(t) = g(\theta, v), \quad v \in V \subseteq R^1 \tag{16}$$

The standard strategy to solve the path following problem is based on the definition of the error state

$$x_e(t) := x(t) - p(\theta(t)) \tag{17}$$

So the path following problem is to find admissible inputs $u(t)$ and $v(t)$ such that the system state $x(t)$ could converge to the reference, i.e. $\lim_{t\to\infty} x_e(t) = 0$.

To solve the above considered path following problem, a nonlinear model predictive framework is proposed.

A. Online Optimization Problem

As a standard framework in NMPC, at each sampling time instant t_k, the following optimization control problem is solved repeatedly:

Problem 1

$$\underset{\bar{u}(\cdot), \bar{v}(\cdot), \theta(\cdot)}{\text{minimize}} J(\bar{x}(t), \bar{u}(t)) \tag{18a}$$

subject to

$$\dot{\bar{x}}(t) = f(\bar{x}(\tau), \bar{u}(\tau)) \;\; \bar{x}(t_k) = x(t_k) \tag{18b}$$

$$\dot{\theta}(\tau) = g(\theta(\tau), v(\tau)) \tag{18c}$$

$$\forall \tau \in [t_k, t_k + T_P]: \bar{x}(\tau) \in X, \quad \bar{u}(\tau) \in U \tag{18d}$$

$$\theta(\tau) \in \hat{\theta}, \quad v(\tau) \in V \tag{18e}$$

$$x_e(\tau) = \bar{x}(\tau) - p(\theta(\tau)) \tag{18f}$$

$$x_e(t_k + T_P) \in \Omega \tag{18g}$$

with

$$J(\bar{x}(t), \bar{u}(t)) = \int_{t_k}^{t_k + T_P} F(x_e(\tau), u(\tau))\, d\tau + E(x_e(t_k + T_P)) \tag{19}$$

where $J(\cdot,\cdot)$ is the cost functional, $\bar{x}$ and $\bar{u}$ represent the predicted system state and input. The term $F(\cdot,\cdot)$ is the stage cost function, which is continuous and positive definite. The time T_P is the prediction horizon. $E(x_e(t_k + T_P))$ and (18g) are the terminal penalty and terminal constraint respectively, which are adopted to guarantee the recursive feasibility of the optimization problem and asymptotic convergence to the reference path. The terminal region Ω denotes that the predicted state $\bar{x}$ has to be restricted inside it at the end of each prediction.

Besides the standard constraints (18b), (18d) and (18g), the further extra constraints (18c) and (18e) describe the evolution of the reference path. The reference path is parameterized in its path parameter $\theta(t)$. Although the $\theta(t)$ is time dependent, its time evolution isn't given a priori and should be determined in the controller to achieve error minimization, including its initial value. The virtual input v controls the evolution of path parameter θ, which is an extra determined variable in the NMPC controller. So the open loop optimization problem (18) is an extended NMPC.

The algorithm of the proposed NMPC scheme at each sampling instance consists of four parts: First, measure the current state $x(t)$ at time t_k . Second, solve the optimization problem (18) in order to achieve a feasible solution $u(\cdot)$, $v(\cdot)$ and $\theta(\cdot)$ for $\tau \in [t_k, t_k + T_P]$. Please notice that to solve (18c), the initial condition $\theta(t_k)$ is necessary for all sampling instances. If the starting value $\theta(t_k)$ is not given a priori, we need to calculate an initial path point parameter θ_0 by solving

$$\underset{\theta_0}{\text{minimize}} \left\| x_0 - p(\theta_0) \right\| \tag{20}$$

Then, the input signal $u(\tau)$, $\tau \in [t_k, t_k + \Delta]$ is applied to the system, where Δ is the sampling time. Finally, use the input value $v(\cdot)$ and the corresponding value of $\theta(\cdot)$ to update the path parameter θ for $\tau \in [t_k, t_k + \Delta]$, and set $t_k = t_k + \Delta$ for going back to the first step.

B. Convergence of NMPC for Path Following

Next sufficient conditions for convergence of the proposed NMPC scheme will be given based on well-known results. A few assumptions should be followed as in [3, 19] at first.

Assumption 1 The reference path P is contained in the state constraint set, i.e. $P \subseteq X$ and $p(0) = 0$.

Assumption 2 Admissible inputs $u \in U$ and $v \in V$ exist such that the state $x \in X$ and path parameter $\theta \in \hat{\theta}$ satisfy $\dot{x}_e(t) = 0$ if $x_e(t)$ equals zero.

Assumption 3 The input constraint $U \subseteq R^m$ is compact and convex with the origin in its interior. The state constraint $X \subseteq R^n$ is closed and connected and contains the origin.

Assumption 4 For the arbitrary initial conditions $x_0 \in X$ and piecewise right continuous input $u \in U$, system (14) has an unique continuous solution.

Assumption 5 $g(\theta, v)$ in (16) has the same requirements as $f(x,u)$. Besides, $\forall v \in V$ and $\forall \theta \in \hat{\theta}$ such that $g(\theta, v) > 0$.

Hence the convergence and feasibility of the propose NMPC scheme can be given by the following theorem.

Theorem 1 (Feasibility and Convergence of NMPC)

Consider the given path following problem for system (14), and suppose that A1-A5 are satisfied. Assume that:

(i) The terminal penalty $E(x_e)$ is continuously differentiable and positive semi-definite. Besides, $E(0) = 0$. The terminal region $\Omega \subseteq X$ is closed and connected with the origin inside its interior. There exists a terminal locally asymptotically controller $k(x_e) \subseteq U$ for all $x_e \in \Omega$, and $k(0) = 0$ holds.

(ii) The terminal feedback control law $u = k(x_e) \subseteq U$ and $E(x_e)$ satisfy

$$\frac{\partial E(x_e)}{\partial x_e} \dot{x}_e + F(x_e, u) \le 0 \tag{21}$$

for all $x_e \in \Omega$, where $\dot{x}_e = \dot{x} - [p(\theta)]' = f(x,u) - \frac{\partial p}{\partial \theta} g(\theta, v)$.

(iii) The proposed NMPC open-loop optimal control problem (18) is feasible at initial time instant t_0 .

Then, the optimal control problem (18) has a feasible solution for all time instants. And the path following error x_e can converge to zero when $t \to \infty$, i.e. $\lim_{t \to \infty} x_e(t) = 0$.

The proof of the Theorem 1 has been explained explicitly in [4] and [19]. The main issues of the proof are summarized briefly here. First, the condition (ii) in Theorem 1 requires a cost decrease for all $x_e \in \Omega$. If the value function of the optimal control problem (18) is defined as a Lyapunov function, one can easily prove that the NMPC value function is nonincreasing, which implies the system is asymptotically stable. Second, the recursive feasibility is guaranteed by concatenating optimal input with a terminal control law $k(x_e)$.

As we can see from the analysis above, finding the suitable terminal penalty $E(\cdot)$, terminal region Ω , and terminal controller $k(\cdot)$, which are desirable for the requirements from Theorem 1, is quite an important part of the NMPC controller design. Unfortunately, it's usually difficult to calculate these terminal ingredients as mentioned before. To solve this problem, let us review the form of the terminal region Ω defined as follows in the standard NMPC framework first

$$\Omega := \{ x_e \in R^n \, | E(x_e) \le \alpha \} \tag{22}$$

where $\alpha > 0$. Ω can be seen as a neighbourhood of the error state $x_e = 0$ by (22). And (18g) is essentially an inequality constraint. Although the calculation of the terminal region is difficult, it's easy to get the information of the reference state in the controller design, so we can take full advantage of the reference path. Redefine the terminal region as follows

$$\Omega := \{0\} \tag{23}$$

Equation (23) means that the path following error x_e should be forced to equal zero at the end of each prediction. In other words the terminal region is chosen to equal the path. At this moment, (18g) is an equality constraint by replacing (22) with (23). By using the structure of (21), the following analysis can show how the given terminal region (23) guarantees asymptotic convergence to the reference path.

Assume that the stage cost function for the optimal control problem (18) is given by

$$F = x_e^T Q x_e + u^T R u \tag{24}$$

where Q, R are the strictly positive weight matrices. If the open-loop optimal control problem (18) is feasible at initial time instant t_0, due to the Assumption 1 and Assumption 2 the system (14) can follow the reference path if $x_e = 0$ .Then the stage cost function (24) only depends on u since $x = p(\theta)$ holds. Moreover, $F = k(x_e)^T R k(x_e) = 0$ for $k(x_e) = k(0) = 0$. As long as we just choose the terminal penalty as $E(\cdot) = 0$, the inequality condition (21) is satisfied. Besides, $E(\cdot) = 0$ and the terminal region defined as (23) satisfy all requirements from Theorem 1. Therefore, the path following error x_e can converge to zero asymptotically for the closed-loop system given by (14), (18) and (23).

IV. SIMULATIONS

In order to verify the effectiveness of the proposed control scheme in Section III, we apply it to the wheeled mobile robot given by (9). A quadratic function (24) is chosen as the stage cost function, with the weight matrices $Q = diag(0.5, 0.5, 0.5)$ and $R = diag(0.5, 0.5)$. The parameters of the mobile robot are $r = 80mm, 2b = 460mm, d = 750mm$, the requirements for control signal are $0 \le v \le 3\ m/s, -3.5 \le w \le 3.5\ rad/s$. The reference input $v_R = 0.7m/s$. $T_P = 10$ and the sampling time is $\Delta = 0.2s$. The time evolution of the path parameter θ is described by the relationship between the velocity and the curve length from the last desired point to the current desired point, that is, (16) is determined by $[p(\theta)]' = \frac{\partial p}{\partial \theta}\dot{\theta} = v$.

We adopt three kinds of reference paths to test the control performance of the proposed scheme. The terminal region is chosen to equal the path, and the other two terminal ingredients are equal to zero in each test. The initial state is the same one $(-0.4, -0.8, \pi/2)$. The first path is an eight-shaped curve with changes in curvature

$$x_R = 1.8\sin\theta, \qquad y_R = 1.2\sin(2\theta) \tag{25}$$

The simulation results are shown as Fig.2 to Fig.4. As shown from the results, the proposed scheme made the robot reach the reference path, besides the real path could follow the reference eight-shaped curve well with the admissible inputs, and the error of each state could converge to zero. The control inputs can be smoothed by increasing the weight matrix R.

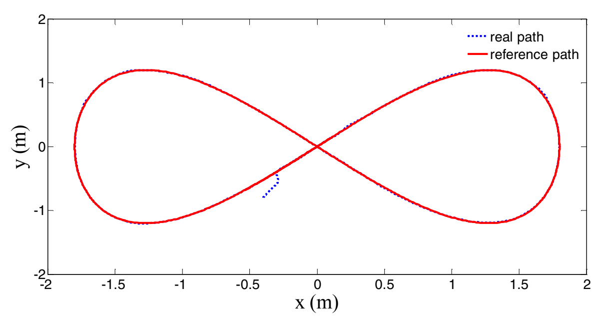

Fig. 2 The reference path and real path

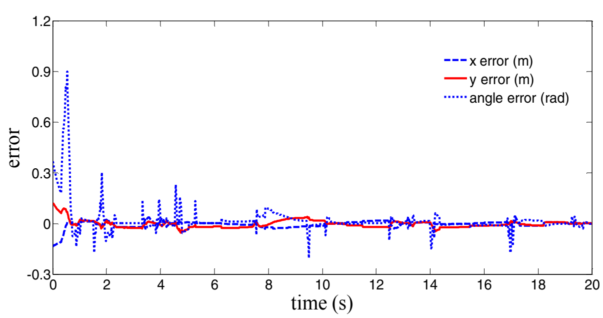

Fig. 3 The states error under NMPC scheme

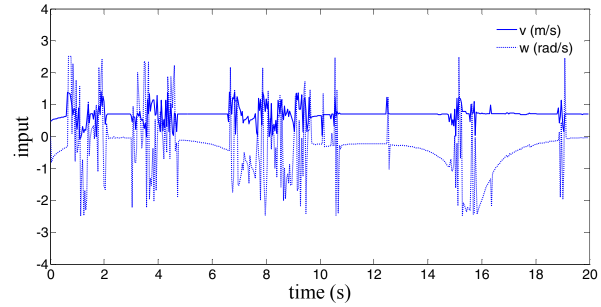

Fig. 4 The control inputs of the NMPC scheme

The second path has the straight part, which is given by

$$x_R = 1.8\sin\theta,$$
$$y_R = \begin{cases} 1.2\sin(2\theta), & 0 \le \theta \le \theta_k \\ 1, & \theta > \theta_k \end{cases} \tag{26}$$

where θ_k is a specific chosen path parameter. The respective simulation results are shown as Fig.5 to Fig.7. In the second simulation, the NMPC controller could also drive the robot to follow the reference path with admissible inputs. Before the robot entered the straight road, it had already followed the path, so the error state could converge to zero faster. The computation speed was faster than the first test.

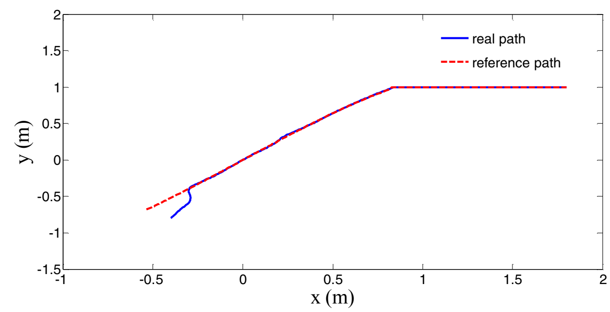

Fig. 5 The reference path and real path

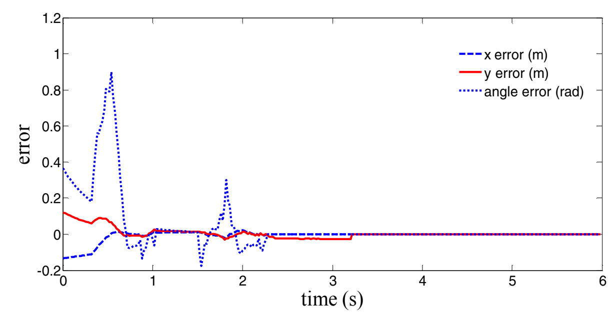

Fig. 6 The states error under NMPC scheme

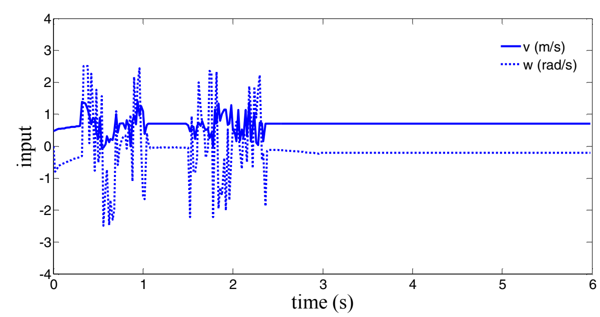

Fig. 7 The control inputs of the NMPC scheme

The two simulations show that the robot has the ability to follow the reference path with high accuracy once it steps on the reference. The third path is a circle, which is defined as

$$x_R = 1.2\cos\theta, \qquad y_R = 1.2\sin\theta \tag{27}$$

The simulation results are shown as Fig.8 to Fig.10.

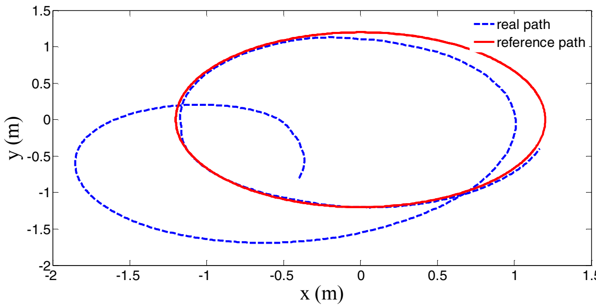

Fig. 8 The reference path and real path

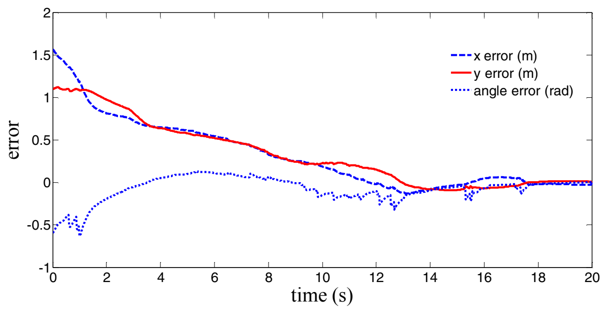

Fig. 9 The states error under NMPC scheme

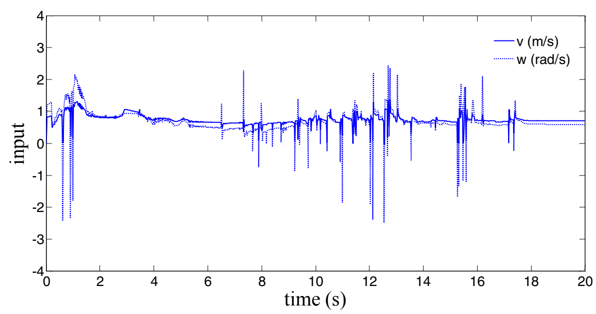

Fig. 10 The control inputs of the NMPC scheme

Although the states of the robot had errors at the preliminary following, the real path finally could converge to the reference circle path, the error states could converge to zero. And the admissible control input approached the reference input at last. As we can see from the three simulations above, it is a feasible way to consider the reference path P as terminal region.

V. Conclusions

After describing the kinematics model of a wheeled mobile robot, a general NMPC scheme was implemented to the path following control of the wheeled mobile robot, where the reference path was chosen as the terminal region. Asymptotic convergence of the proposed scheme and recursive feasibility of the optimization problem were obtained by means of the well known stability results. Especially, the time evolution of the path parameter as well as its initial value was determined online. At last the proposed scheme was applied to verify its effectiveness. The proposed control strategy guarantees that the following error can converge to zero asymptotically, and offers a scheme to solve the path following problem.

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